

## Discrete Mathematics

Question (5):  $a_n = \overset{c_1}{10}a_{n-1} + \overset{c_2}{29}a_{n-2}, \quad n \geq 2$   
 $a_0 = 0, \quad a_1 = 1$

$$\Rightarrow c_1 = 10, \quad c_2 = 29 \Rightarrow \overset{a}{1}r^2 - \overset{b}{10}r - \overset{c}{29} = 0 \quad \text{"C.E."}$$

$$\Rightarrow r_1 = 5 + 3\sqrt{6}, \quad r_2 = 5 - 3\sqrt{6} \quad \left( r = \frac{10 \pm \sqrt{100 - 4 \times 1 \times -29}}{2 \times 1} \right)$$

$$\Rightarrow a_n = \alpha_1 (5 + 3\sqrt{6})^n + \alpha_2 (5 - 3\sqrt{6})^n$$

$$\Rightarrow a_0 = \alpha_1 + \alpha_2 = 0 \quad \longrightarrow \quad \alpha_2 = -\alpha_1$$

$$\Rightarrow a_1 = \alpha_1 (5 + 3\sqrt{6}) + \alpha_2 (5 - 3\sqrt{6}) = 1$$

$$\Rightarrow \alpha_1 (5 + 3\sqrt{6}) - \alpha_1 (5 - 3\sqrt{6}) = 1 \Rightarrow \alpha_1 (6\sqrt{6}) = 1 \Rightarrow \alpha_1 = \frac{1}{6\sqrt{6}}$$

$$\Rightarrow \alpha_1 = \frac{1}{6\sqrt{6}}, \quad \alpha_2 = \frac{-1}{6\sqrt{6}}$$

$$\Rightarrow \text{Thus, } a_n = \left( \frac{1}{6\sqrt{6}} \right) (5 + 3\sqrt{6})^n - \left( \frac{1}{6\sqrt{6}} \right) (5 - 3\sqrt{6})^n, \quad n \geq 2$$